**Exercise 2.**

The VAR model in matrix notation is:

Equivalently,

The **likelihood function of the model** is

where .

The **pdf of the joint matrix-variate normal-inverse Wishart prior distribution for parameter matrices and**  is

The **parameters determining the prior distribution matrices** are specified as the following:

* Macroeconomic variables are unit-root nonstationary and are well-characterised by a multivariate random walk process.

, where and is a vector of ones

* This is to determine the prior shrinkage, where determines the overall shrinkage level for autoregressive slopes and determines the overall shrinkage for the constant term.

,

* These are to express the stylised facts that the data are little informative about the values of the constant term and strongly favour the unit-root hypothesis.

, where

* This is the degrees of freedom, simply set to such that .

* Number of variables

* To capture the seasonality in quarterly data.

* Sample size

**Exercise 3.**

The **joint conditional posterior distribution of matrices and given the data matrices and and hyper-parameters and**  is

where

Therefore,

**Exercise 4.**

We propose that the **conditionally conjugate prior distributions for the hyper-parameters and** , follow an independent inverse gamma 2 and gamma distribution such that

Hence the pdf is such that

**Exercise 5.**

Since

where is not related to and ,

hence

The **full conditional posterior distribution for the hyper-parameter**  is

where

Therefore,

The **full conditional posterior distribution for the hyper-parameter**  is

where

Therefore,

**Exercise 6**

The **Gibbs sampler** for the estimation of the parameters of the model using the full conditional posterior distributions is presented as the following.

Initialise at and at .

At each iteration :

1. Sample , where

2. Sample , where

3. Jointly sample , where

Repeat steps 1. 2. and 3. times.

Discard the first draws for the sake of convergence.

Output is a sample of draws from the joint posterior distribution ,

For and .